International Journal of Physical Sciences

Volume 9 Number 1 16 January, 2014
ISSN 1992-1950

Academic Journals

ABOUT IJPS

The **International Journal of Physical Sciences (IJPS)** is published weekly (one volume per year) by Academic Journals.

International Journal of Physical Sciences (IJPS) is an open access journal that publishes high-quality solicited and unsolicited articles, in English, in all Physics and chemistry including artificial intelligence, neural processing, nuclear and particle physics, geophysics, physics in medicine and biology, plasma physics, semiconductor science and technology, wireless and optical communications, materials science, energy and fuels, environmental science and technology, combinatorial chemistry, natural products, molecular therapeutics, geochemistry, cement and concrete research, metallurgy, crystallography and computer-aided materials design. All articles published in IJPS are peer-reviewed.

Submission of Manuscript

Submit manuscripts as e-mail attachment to the Editorial Office at: ijps@academicjournals.org. A manuscript number will be mailed to the corresponding author shortly after submission.

For all other correspondence that cannot be sent by e-mail, please contact the editorial office (at ijps@academicjournals.org).

The International Journal of Physical Sciences will only accept manuscripts submitted as e-mail attachments.

Please read the **Instructions for Authors** before submitting your manuscript. The manuscript files should be given the last name of the first author.

Editorial Board

Instructions for Author

Electronic submission of manuscripts is strongly encouraged, provided that the text, tables, and figures are included in a single Microsoft Word file (preferably in Arial font).

The **cover letter** should include the corresponding author's full address and telephone/fax numbers and should be in an e-mail message sent to the Editor, with the file, whose name should begin with the first author's surname, as an attachment.

Article Types

Three types of manuscripts may be submitted:

Regular articles: These should describe new and carefully confirmed findings, and experimental procedures should be given in sufficient detail for others to verify the work. The length of a full paper should be the minimum required to describe and interpret the work clearly.

Short Communications: A Short Communication is suitable for recording the results of complete small investigations or giving details of new models or hypotheses, innovative methods, techniques or apparatus. The style of main sections need not conform to that of full-length papers. Short communications are 2 to 4 printed pages (about 6 to 12 manuscript pages) in length.

Reviews: Submissions of reviews and perspectives covering topics of current interest are welcome and encouraged. Reviews should be concise and no longer than 4-6 printed pages (about 12 to 18 manuscript pages). Reviews are also peer-reviewed.

Review Process

All manuscripts are reviewed by an editor and members of the Editorial Board or qualified outside reviewers. Authors cannot nominate reviewers. Only reviewers randomly selected from our database with specialization in the subject area will be contacted to evaluate the manuscripts. The process will be blind review.

Decisions will be made as rapidly as possible, and the journal strives to return reviewers' comments to authors as fast as possible. The editorial board will re-review manuscripts that are accepted pending revision. It is the goal of the IJPS to publish manuscripts within weeks after submission.

Regular articles

All portions of the manuscript must be typed double-spaced and all pages numbered starting from the title page.

The Title should be a brief phrase describing the contents of the paper. The Title Page should include the authors' full names and affiliations, the name of the corresponding author along with phone, fax and E-mail information. Present addresses of authors should appear as a footnote.

The Abstract should be informative and completely self-explanatory, briefly present the topic, state the scope of the experiments, indicate significant data, and point out major findings and conclusions. The Abstract should be 100 to 200 words in length.. Complete sentences, active verbs, and the third person should be used, and the abstract should be written in the past tense. Standard nomenclature should be used and abbreviations should be avoided. No literature should be cited.

Following the abstract, about 3 to 10 key words that will provide indexing references should be listed.

A list of non-standard **Abbreviations** should be added. In general, non-standard abbreviations should be used only when the full term is very long and used often. Each abbreviation should be spelled out and introduced in parentheses the first time it is used in the text. Only recommended SI units should be used. Authors should use the solidus presentation (mg/ml). Standard abbreviations (such as ATP and DNA) need not be defined.

The Introduction should provide a clear statement of the problem, the relevant literature on the subject, and the proposed approach or solution. It should be understandable to colleagues from a broad range of scientific disciplines.

Materials and methods should be complete enough to allow experiments to be reproduced. However, only truly new procedures should be described in detail; previously published procedures should be cited, and important modifications of published procedures should be mentioned briefly. Capitalize trade names and include the manufacturer's name and address. Subheadings should be used. Methods in general use need not be described in detail.

Results should be presented with clarity and precision. The results should be written in the past tense when describing findings in the authors' experiments. Previously published findings should be written in the present tense. Results should be explained, but largely without referring to the literature. Discussion, speculation and detailed interpretation of data should not be included in the Results but should be put into the Discussion section.

The Discussion should interpret the findings in view of the results obtained in this and in past studies on this topic. State the conclusions in a few sentences at the end of the paper. The Results and Discussion sections can include subheadings, and when appropriate, both sections can be combined.

The Acknowledgments of people, grants, funds, etc should be brief.

Tables should be kept to a minimum and be designed to be as simple as possible. Tables are to be typed double-spaced throughout, including headings and footnotes. Each table should be on a separate page, numbered consecutively in Arabic numerals and supplied with a heading and a legend. Tables should be self-explanatory without reference to the text. The details of the methods used in the experiments should preferably be described in the legend instead of in the text. The same data should not be presented in both table and graph form or repeated in the text.

Figure legends should be typed in numerical order on a separate sheet. Graphics should be prepared using applications capable of generating high resolution GIF, TIFF, JPEG or Powerpoint before pasting in the Microsoft Word manuscript file. Tables should be prepared in Microsoft Word. Use Arabic numerals to designate figures and upper case letters for their parts (Figure 1). Begin each legend with a title and include sufficient description so that the figure is understandable without reading the text of the manuscript. Information given in legends should not be repeated in the text.

References: In the text, a reference identified by means of an author‘s name should be followed by the date of the reference in parentheses. When there are more than two authors, only the first author‘s name should be mentioned, followed by ’et al‘. In the event that an author cited has had two or more works published during the same year, the reference, both in the text and in the reference list, should be identified by a lower case letter like ’a‘ and ’b‘ after the date to distinguish the works.

Examples:

Abayomi (2000), Agindotan et al. (2003), (Kelebeni, 1983), (Usman and Smith, 1992), (Chege, 1998; 1987a,b; Tijani, 1993,1995), (Kumasi et al., 2001)

References should be listed at the end of the paper in alphabetical order. Articles in preparation or articles submitted for publication, unpublished observations, personal communications, etc. should not be included in the reference list but should only be mentioned in the article text (e.g., A. Kingori, University of Nairobi, Kenya, personal communication). Journal names are abbreviated according to Chemical Abstracts. Authors are fully responsible for the accuracy of the references.

Examples:

Ogunseitan OA (1998). Protein method for investigating mercuric reductase gene expression in aquatic environments. Appl. Environ. Microbiol. 64:695-702.

Gueye M, Ndoye I, Dianda M, Danso SKA, Dreyfus B (1997). Active N2 fixation in several Faidherbia albida provenances. Ar. Soil Res. Rehabil. 11:63-70.

Charnley AK (1992). Mechanisms of fungal pathogenesis in insects with particular reference to locusts. In: Lomer CJ, Prior C (eds) Biological Controls of Locusts and Grasshoppers: Proceedings of an international workshop held at Cotonou, Benin. Oxford: CAB International, pp 181-190.

Mundree SG, Farrant JM (2000). Some physiological and molecular insights into the mechanisms of desiccation tolerance in the resurrection plant Xerophyta viscasa Baker. In Cherry et al. (eds) Plant tolerance to abiotic stresses in Agriculture: Role of Genetic Engineering, Kluwer Academic Publishers, Netherlands, pp 201-222.

Short Communications

Short Communications are limited to a maximum of two figures and one table. They should present a complete study that is more limited in scope than is found in full-length papers. The items of manuscript preparation listed above apply to Short Communications with the following differences: (1) Abstracts are limited to 100 words; (2) instead of a separate Materials and Methods section, experimental procedures may be incorporated into Figure Legends and Table footnotes; (3) Results and Discussion should be combined into a single section.

Proofs and Reprints: Electronic proofs will be sent (e-mail attachment) to the corresponding author as a PDF file. Page proofs are considered to be the final version of the manuscript. With the exception of typographical or minor clerical errors, no changes will be made in the manuscript at the proof stage.

International Journal of Physical Sciences

Table of Contents: Volume 9 Number 1, 16 January, 2014

ARTICLES

academicJournals

Vol. 9(1), pp. 1-6, 16 January, 2014
DOI: 10.5897/IJPS10.672
ISSN 1992 - 1950 © 2014 Academic Journals
http://www.academicjournals.org/IJPS

International Journal of Physical Sciences

Full Length Research Paper

Study of optical constants in Se- In –Pb thin films of chalcogenide glasses

Preeti Dwivedi[1] and Rita Awasthi[2]

[1]Department of Physics, V.S.S.D. College, Kanpur, India.
[2]Department of Chemistry, Brahmanand College, Kanpur, India.

Accepted 27 December, 2013

Bulk glasses of Se_{75} In_{25-x} Pb_x (when n = 0, 4, 6, 10) systems and thin films have been prepared by the conventional melt quenching technique and by thermal evaporation technique, respectively. The optical constants (absorption coefficient (α), refractive index (n), extinction coefficient (k)) have been studied for Se-In-Pb thin films as a function of photon energy in the wavelength range (300-900 nm). Spectral dependence of transmittance (T) and reflectance (R) in the spectral range (300-900 nm) of glass Se-In-Pb have also been observed at various concentrations of Te and Pb. The effect of compositional variation on some optical constant has been observed and discussed in the present investigation.

Key words: optical constants, thin films, chalcogenide glasses.

INTRODUCTION

Chalcogenide glasses have been studied intensively in the last few years owing to their interesting properties and technological applications (Al-Ghamdi, 2006). They are materials used in optical and photonic devices. They have good electro-optic, thermo-optic, magneto-optic, acousto-optic properties, high refractive index (Barman and Sharma, 2013) and I R transparency (Zakery and Elliott, 2003). A good deal of information regarding the synthesis and opto–electro-structural properties of thin films of lead chalcogenides have been revealed by Kumar et al. (2005). The development of laser technology had opened up new application for narrow gap lead salts and their alloys. The polycrystalline thin films were deposited onto optically plane and chemically clean glass substrates by vacuum evaporation technique. The films were thin, uniform, smooth and tightly adherent to the substrates. Optical absorption spectroscopy, X-ray diffraction technique and current–voltage characteristics method were used to characterize the films.

Chalcogenide glasses material heating to melting is the form of a liquid and at some instance; it freezes the position of every atom by quenching. Even in this freezed position, they retain short range order and the position of the nearest neighbour remains nearly the same. These glasses are useful in the preparation of passive devices like lenses, windows, fibers (Books et al., 1999; Hoede et al., 2001), active devices like laser fiber amplifiers (Schweitzer et al., 2001) and non-linear components (Dixit and Vijaya, 2001). The present work has been carried out in order to investigate some optical properties of Se_{75} In_{25-x} Pb_x system. The effect of incorporation of lead in Se-In alloy can change the structural and optical properties of the system.

METHODOLOGY

Glassy alloys of Se-In-Pb (x=0, 4, 6, 10) have been prepared by melt-quenching techniques. Extra pure materials (99.99%) were weighed according to their atomic percentage and sealed in 6 cm long quartz ampoules with diameter 6 mm in the vacuum of 10^{-5} torr. The sealed ampoules were kept in a furnace at 700°C and were held at the same temperature for about 10 h. The temperature of the furnace was raised slowly at a rate of 3-4°C/min. During heating, all the ampoules were constantly rocked for better homogenization of the alloys. This was achieved with the help of a

*Corresponding author. E-mail: rita_awasthi2000@yahoo.co.in

Table 1. Variation of Absorption coefficient (α) with photon energy in a se_{75} In_{25-x} Pbx thin films at room temperature.

S/N	Photon energy E (eV)	Absorption coefficient (α) in cm^4			
		$Se_{75}In_{25}$ Pb_0	$Se_{75}In_{21}$ Pb_4	$Se_{75}In_{19}$ Pb_6	$Se_{75}In_{15}$ Pb_{10}
1	1.377	15555.556	7777.778	N/A	3888.889
2	1.458	15555.556	7777.778	3888.889	3888.889
3	1.549	14000.00	6222.222	4666.667	4666.667
4	1.906	22555.556	6222.222	17888.889	23333.333
5	2.065	38888.889	15555.556	27222.222	38888.889
6	2.253	62222.222	29555.556	49777.778	56000.000
7	2.478	79333.333	42000.000	73888.889	70000.000
8	2.753	101111.111	56000.000	101111.111	101111.111
9	3.540	112777.778	88666.667	163333.333	136111.111
10	4.130	217777.778	217777.778	256666.667	256666.667

ceramic rod to which all the ampoules were tucked away in the furnace. Rotating the same rod after every five minutes interval of time did this. Thereafter, the obtained melts were cooled rapidly by removing the ampoules from the furnace and quenching into the ice cold water. The quenched samples were taken out by breaking the ampoules. The amorphous nature of the glassy alloys was identified by X - Ray diffraction. Thin films of glassy alloys of thickness (~300 nm) were prepared by thermal evaporation technique in which substrate was kept at room temperature at a base pressure of 10^{-4} torr using the molybdenum boat; the film was kept inside the deposition chamber for 24 h to achieve the stable equilibrium (Ogusk et al., 2004). The thickness of the film was measured by a single crystal thickness monitor. In order to measure the optical absorption reflection, transmission as a function of wavelength (300-900 nm), a double UV/VIS/NIR computerized spectrophotometer (JASCO V-500) has been used.

RESULTS AND DISCUSSION

The value of optical absorption coefficient (α) as a function of incident photon (hν) for deposited thin films of Se_{75} In_{25-x} Pb_x where x= 0, 4, 6, 10 at room temperature are shown in Table 1. The absorption coefficient (α) has been obtained directly from the absorbance against wavelength curves using the relation

$$\alpha = OD/t \quad (1)$$

Where OD is the optical density measured at a given layer thickness (t).

It has been observed in Table 1 that the value of absorption coefficient (α) increases linearly with the increase in photo energy of Se_{75} In_{25-x} Pb_x. According to Mott and Davis (1979), the absorption of light by amorphous solids depends on the energies of the incident photon and on the optical band gap of the material. Thus the behavior may be represented by the following equation.

$$\alpha h\nu = B(h\nu - Eg) \quad (2)$$

Where B is a constant which depends on the transition probability, hν is energy of incident photon, Eg is optical band gap of the material, r is an index which depends on the nature of transition (Hasegawa et al., 1978) where r = 1/2 and r = 2 for allowed direct transition and indirect transition, respectively.

The present system obeys the rule of indirect transition and the relation between optical gap, optical coefficient (α), and the energy hν of the incident photon is given by

$$(\alpha h\nu)^{1/2} = A(h\nu - Eg) \quad (3)$$

Where α obeys - Urbach relation (Urbach, 1953)

$$\alpha = \exp[A(h\nu - h\nu_0)]/kT \quad (4)$$

Where A is a constant of the order of unity, ν_0 is the constant corresponding to the lowest excitonic frequency.

The variation of $(\alpha h\nu)^{1/2}$ with photon energy hν for Se_{75}-In_{25} - x Pb_x films are shown in Figure 1. The value of indirect optical band gap (Eg) has been calculated by taking the intercept on the x-axis. The calculated values of Eg are given in Table 2. It is evident from the table that the value of optical band (Eg) increases from 1.24 to 1.32 with increasing Pb content in Se_{75} In_{25}- x Pb_x. The increase in the Eg with increasing Pb may be due to increase in grain size, the reduction in the disorder, and decrease in density of defect states.

The value of Eg decreases from 1.32 to 1.25 with increasing Pb concentration in Se_{25} In_{25}-x Pb_X sample, since the optical absorption depends on the short range order in the amorphous state and defects associated with it. The decrease in the optical band gap in the present system may be due to reduction of the amount of disorder in the system and increase in the density of defect state. It may be due to (Borgogro et al., 1982) the shift in Fermi level whose position is determined by the distribution of electrons over the localized states (Miller et al., 1988). The values of the variation of $(\alpha h\nu)^{1/2}$ with photon energy (hν) for Se_{75} In_{25}- x Pb x films are given in Table 3.

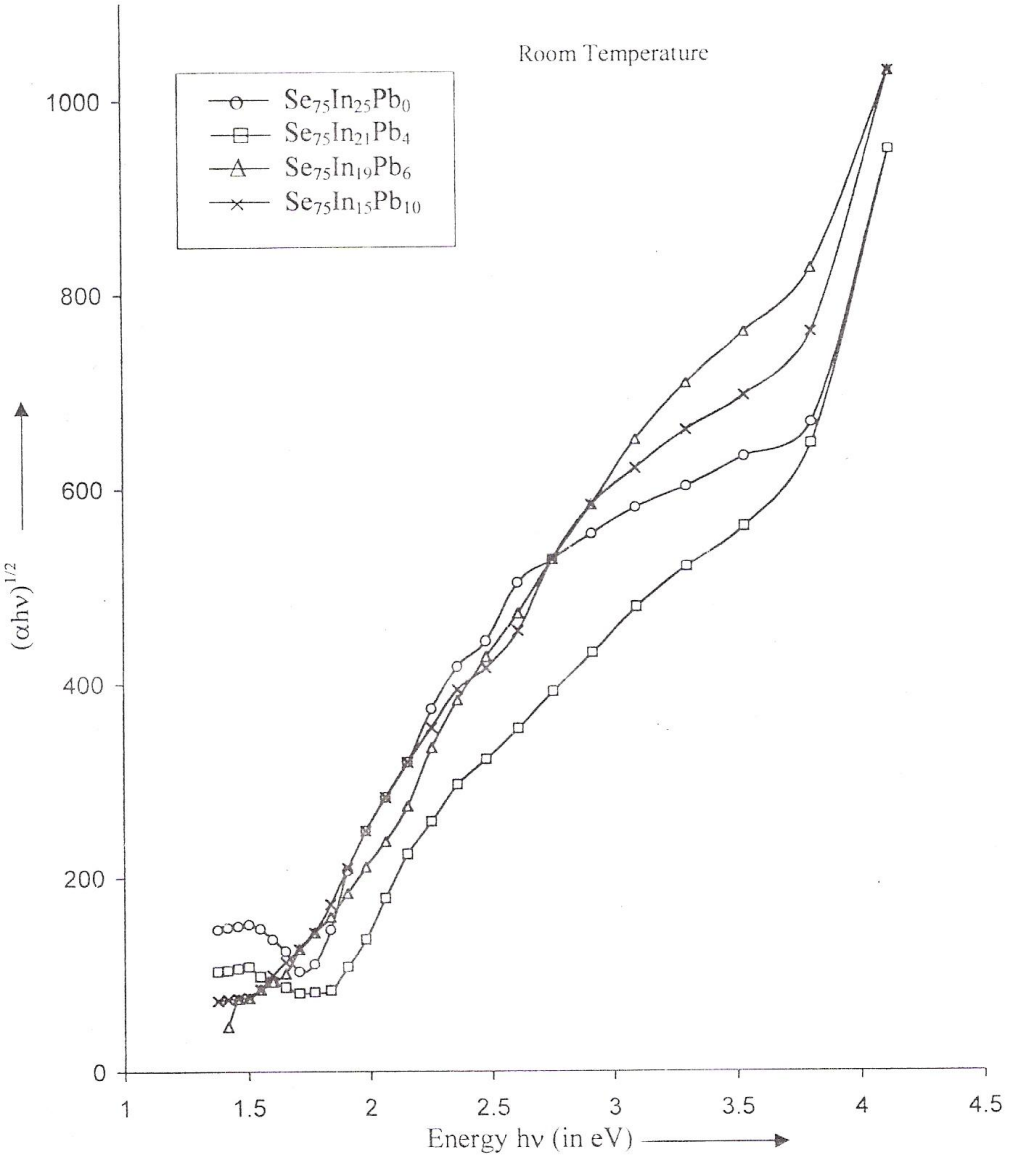

Figure 1. Variation of $(\alpha h\nu)^{1/2}$ with photon energy in a Se_{75} In_{25-x} Pbx thin films at room temperature.

Table 2. Optical parameter in $Se_{75}In_{25-x}Pb_X$ thin film at 700 nm at RT.

X	E_g (eV)	α (cm^{-1}) (10^4)	N	k (10^{-3})	$\varepsilon_r^{'}$	$\varepsilon_r^{''}$	R%	T%
0	1.50	0.7	2.016	0.390	4.067	0.00157	15.63	65.625
2	1.24	0.388	2.575	0.2167	6.630	0.0011	29.16	70.625
6	1.32	1.166	5.279	0.6502	27.868	0.00686	75.62	58.33
10	1.25	1.166	2.660	0.6502	7.080	0.00346	31.25	64.166

The variation of reflectance (R) in percentage for the present system of Se_{75} In_{25-x} Pb_x is shown in Table 4. The table shows that the reflectance (R) increases with the increase in wavelength. Similarly, the transmittance (T) has also been shown for Se_{75} In_{25-x} Pb_x in Table 3. It is evident from the table that the percentage in the transmission increases with respect to wavelength. It is clear from the table that the value of the transmission

Table 3. Variation of $(\alpha h\nu)^{1/2}$ with photon energy in a - se_{75} In_{25-x} Pbx thin films at room temperature.

S/N	Photon energy E (eV)	Absorption coefficient (α) in Cm^4			
		$Se_{75}In_{25}$ Pb_0	$Se_{75}In_{21}$ Pb_4	$Se_{75}In_{19}$ Pb_6	$Se_{75}In_{15}$ Pb_{10}
1	1.377	146.338	103.477	N/A	73.169
2	1.441	150.581	106.477	75.290	75.290
3	1.511	147.250	98.167	85.015	85.015
4	1.588	124.172	87.803	101.386	113.353
5	1.674	111.310	82.966	143.701	143.701
6	1.770	207.351	108.906	184.659	210.896
7	1.877	283.382	179.227	237.095	283.382
8	1.998	374.392	258.032	334.867	355.180
9	2.136	443.382	322.608	427.898	416.485
10	2.294	527.629	392.666	527.629	527.629
11	2.478	580.761	478.404	649.311	620.860
12	2.693	631.849	560.250	760.395	694.142
13	2.950	948.379	948.379	1029.579	1029.579
14	3.261	N/A	N/A	N/A	N/A

N/A, Not available.

Table 4. Variation of reflection (R) in % with wavelength in a-se_{75} In_{25-x} Pbx thin films at room temperature.

S/N	Wave length in nm	Reflection (R) in %			
		$Se_{75}In_{25}$ Pb_0	$Se_{75}In_{21}$ Pb_4	$Se_{75}In_{19}$ Pb_6	$Se_{75}In_{15}$ Pb_{10}
1	900	87.500	62.500	31.250	28.125
2	800	70.833	54.583	60.417	13.750
3	700	15.625	29.167	75.625	31.250
4	600	46.875	14.583	30.208	41.667
5	500	31.667	32.500	41.667	28.125
6	400	30.208	31.250	36.458	31.667
7	300	28.333	22.917	29.167	22.917

Table 5. Variation of transmission (T) in % with wave length in a $se_{75}In_{25-x}$ Pbx thin films at room temperature.

S/N	Wave length in nm	Transmission (T) in %			
		$Se_{75}In_{25}$ Pb_0	$Se_{75}In_{21}$ Pb_4	$Se_{75}In_{19}$ Pb_6	$Se_{75}In_{15}$ Pb_{10}
1	900	40.000	56.250	75.000	82.500
2	800	44.792	61.458	78.125	85.938
3	700	65.625	70.625	58.333	64.167
4	600	14.583	48.958	19.792	21.771
5	500	2.083	15.625	3.125	3.438
6	400	N/A	3.125	0.208	0.229
7	300	N/A	0.208	N/A	N/A

N/A, Not available.

starts decreasing after about 600 nm (Table 5). The refractive index (n) and extinction coefficient (k) have been calculated by using the theory of reflectivity of light for the above sample discussed. According to this theory, the reflectance of light from this film can be expressed in turn of fresnel's coefficient. The reflectivity (Scott et al., 1982) on a interface can be given by:

$$R=[(n-1)^2+k^2]/[(n+1)^2+k^2 \quad (5)$$

$$\alpha=4\pi K/\lambda \quad (6)$$

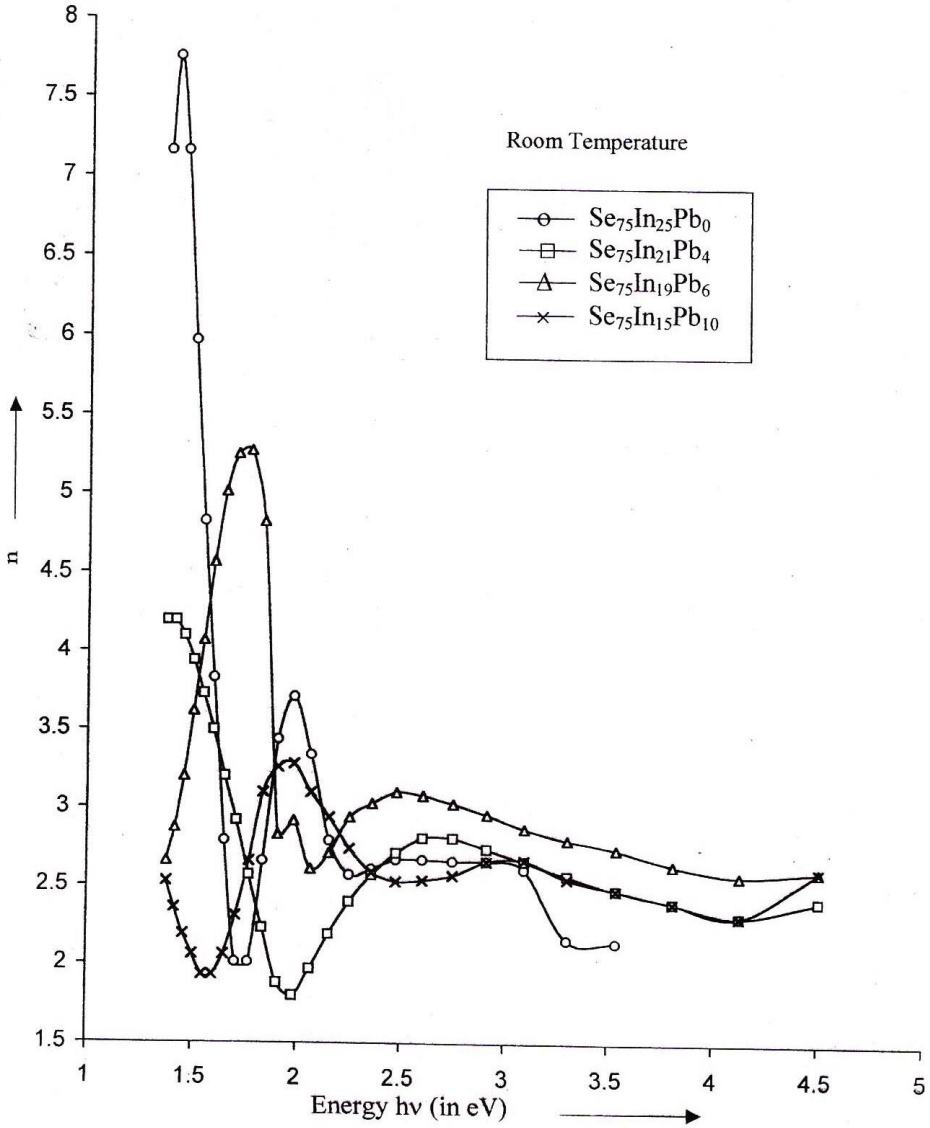

Figure 2. Variation of refractive index (n) with photon energy in a $Se_{75}In_{25-x}$ Pb_x thin films at room temperature.

Where λ is the wavelength

The spectral dependence of refractive index (n) and extinction coefficient (k) for Se_{75} In_{25}- x Pb_x thin films are shown in Figures 2 and 3, respectively. The value of refractive index (n) and the value of extinction coefficient (k) increases with increasing photon energy conclusion.

Conclusion

In this paper we systematically studied the chemical modification of Se_{75} In_{25} - x Pb_x glass system incorporated with Pb. It was found in the optical investigations that the optical band gap increases up to 4% of Pb concentration in Se75 In25-x Pbx; with further increase of Pb content, the optical band gap decreases in the present system. The increase in the optical band gap with increasing Pb content may be due to the increase in the grain size, the reduction in the disorder and decrease in the density of defect states. The value of optical band gap (Eg) decreases at higher Pb concentration. The decrease in the optical band gap in the present system may be due to reduction in the amount of disorder in the system and increase in the density of defect states. From the reflectance and transmittance studies of the film of Se_{75} In_{25} - x Pb_x, it may be concluded that the refractive index (n) decreases, while the value of the extinction coefficient k increases with photon energy.

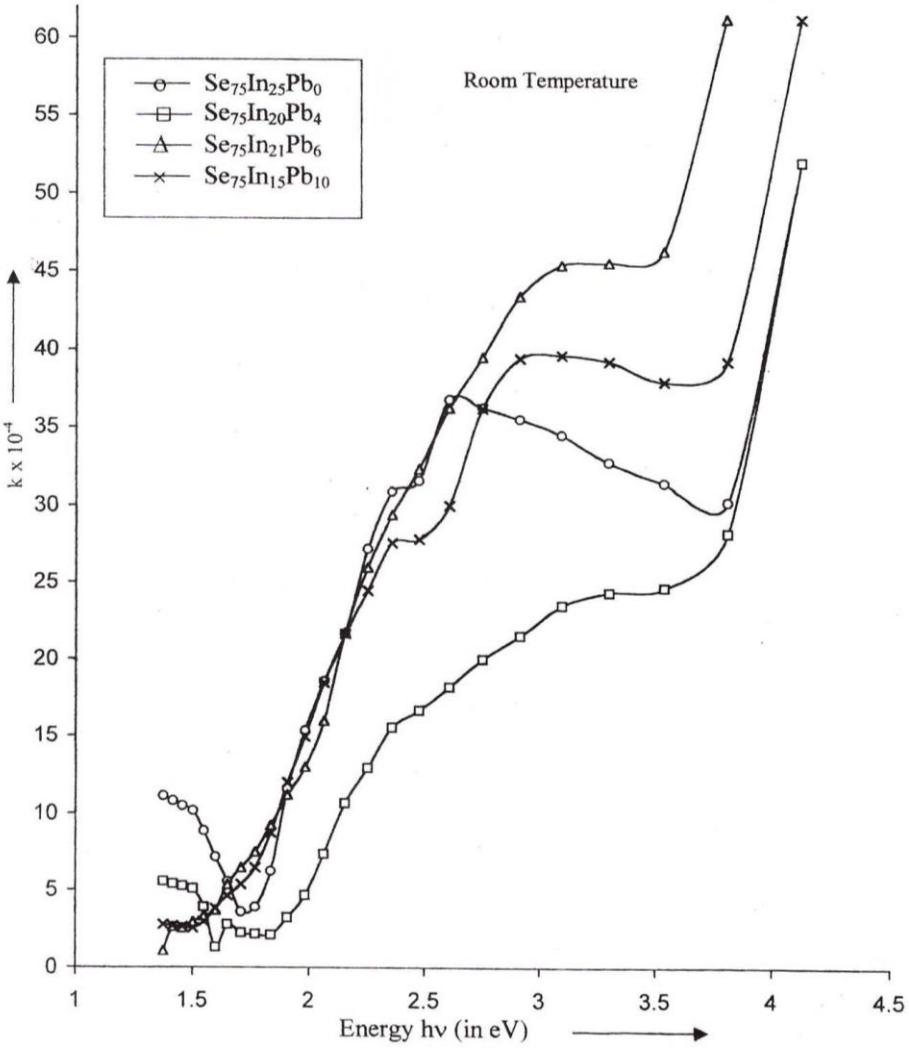

Figure 3. Variation of extinction coefficient (K) with photon energy in a $Se_{75}In_{25-x}$ Pb_x thin films at room temperature.

REFERENCES

Al-Ghamdi AA (2006). Optical band gap and optical constants in amorphous Se96−xTe4Agx thin films. Vacuum. 80(5):400-405.

Barman DB, Sharma P (2013). Optical studies of Se-Bi-Te-Sb thin films by single transmission spectrum. Glass Physics and Chemistry, 39(3):276-278

Books A, Afanasyeva NI, Makhine V, Bruch RF, Mc. Greger B. (1999). FEW-FTIR spectroscopy applications and computer data processing for noninvasive skin tissue diagnostics in vivo. SPIE-The International society for Optical Engineering, 3596:140-151.

Borgogro JP, Lazarides B, Pelletier E (1982). Constants of Inhomogeneous Thin Films. Appl. Optics, 21:4020-4029.

Dixit N, Vijaya R (2001). Chalcogenide glasses for use in photonic switching devices: a detailed analysis. SPIE-The International society for Optical Engineering, 4417:477-482.

Hoede S, Loreal O, Sire O, Turlin B, Boussard-Pledel C, Le. Coq D, Bureau B, Fonteneau G, Pigeon C, Leroyer P, Lucas J (2001). Biological tissue infrared analysis by chalcogenide glass optical fiber spectroscopy. SPIE-The International society for Optical Engineering, 4158:49-56.

Hasegawa S, Yazaki S, Shimizuol T (1978). Solid State Commun. 26:4070.

Kumar S, Khan ZH, Khan MAM, Hussain M (2005). Studies on thin films of lead chalcogenides. Current Appl. Phys. 5(6):561-566.

Miller LS, Walder AJ, Lensell P, Blundell A (1988). Reflectometry Measurement of Optical Parameters of Au/SiO2/Si Films. Thin Solid Films. 165:11.

Mott NF, Davis EA (1979). Electronic Process in Non-Crystalline Materials (1979) (Clarendon, Oxford) P. 428.

Schweitzer T, Goutaland F, Martins E, Hewak DW, Brocklesby WS (2001). Site-selective spectroscopy in dysprosium-doped chalcogenide glasses for 1.3 µm optical fiber amplifiers. J. Optical Soc. Am. B (optical physics), 18(10):1436-1442.

Scott BA, Remier JA, Plecenik RM, Simonyi EE, Reuter W (1982). Low Defect Density Hydrogenated Amorphous Silicon Prepared by Homogeneous Chemical Vapor Deposition. Appl. Phys. Lett. 40:973.

Urbach F (1953). The Long-Wavelength Edge of Photographic Sensitivity and of the Electronic Absorption of Solids. Physics Rev. 92:1324.

Zakery A, Elliott S (2003). Optical properties and applications of chalcogenide glasses: a review. Non-Cryst. Solids, 330:1-3.

academicJournals

Vol. 9(1), pp. 7-12, 16 January, 2014
DOI: 10.5897/IJPS2013.4019
ISSN 1992 - 1950 © 2014 Academic Journals
http://www.academicjournals.org/IJPS

International Journal of Physical Sciences

Full Length Research Paper

A four-point fully implicit method for the numerical integration of third-order ordinary differential equations

Awoyemi D. O.[1], Kayode S. J.[2] and Adoghe L. O.[3]*

[1]Department of Mathematics, Landmark University, Umuaru, Kwara State, Nigeria.
[2]Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria.
[3]Department of Mathematics, Ambrose Alli University, Ekpoma, Edo State, Nigeria.

Accepted 13 January, 2013

In this paper, we derived a continuous linear multistep method (LMM) with step number k = 4 through collocation and interpolation techniques using power series as basis function for approximate solution. An order-seven scheme is developed which was used to solve the third-order initial value problems (IVPS) in ordinary differential equation without first reducing to a system first-order. Taylor's series algorithm of the same order was developed to implement our method. The result obtained was compared favourably with existing methods.

Key words: Continuous collocation, multistep methods, interpolation, third-order, power series.

INTRODUCTION

Linear multistep methods (LMMs) are very popular for solving first-order initial value problems (IVPS). Conventionally, they are used to solve higher order ordinary differential equations by first reducing them to a system of first-order. This approach has been extensively discussed in Bruguano and Trigiante (1998), Lambert (1973) and Fatunla (1988). However, the method of reducing to a system first-order has some serious drawback which includes wastage of human effort and computer time (Butcher, 2003).

The general k-step method or LMM of step number k is as given in Lambert (1973).

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} \quad (1)$$

Where α_j and β_j are uniquely determined and $\alpha_0 + \beta_0 \neq 0$, $\alpha_k = 1$

The LMM in Equation (1) generates schemes discrete schemes which are used to solve first-order ordinary differential equations. Various form of this LMM has been developed (Awoyemi, 2003; Henrici, 1962; Lambert, 1973; Fatunla, 1988). Other researchers have introduced the continuous LMM using the continuous collocation and interpolation approach. This has led to the development of continuous LMM of form

$$Y(x) = \sum_{j=0}^{k} \alpha_j(t) y_{n+j} + h \sum_{j=0}^{k} \beta_j(t) f_{n+j} \quad (2)$$

α_j and β_j are expressed as continuous functions of t and are at least differentiable once.

The introduction of continuous collocation methods as against the discrete schemes enhances better global error estimation and ability to approximate solution at all interior points. In this study, we shall develop continuous multistep collocation method for the solution of third-order ordinary differential equations using power series as the

*Corresponding author. E-mail: adoghelarry@gmail.com.

basis function.

Power series collocation

In Awoyemi (1999) and Onumanyi et al. (1993), some continuous LMM of the type in Equation (2) were developed using the collocation function of form:

$$Y(x) = \sum_{j=0}^{k} \alpha_j x^j \tag{3}$$

However, Adeniyi and Alabi (2006) used Chebyshev polynomial function of the form

$$Y(x) = \sum_{j=0}^{M} \alpha_j T_j(x)\left(\frac{x - x_k}{h}\right) \tag{4}$$

Where $T_j(x)$ are some Chebyshev function to develop continuous LMM.

Power series is proposed as basis function for deriving continuous LMM based on the property of analytic function that given the Taylor's polynomial of the form:

$$p(x) = \sum_{j=0}^{n} \frac{y^r(x_0)(x - x_n)^r}{r!} \tag{5}$$

The approximate function p(x) reduces to Y(x) as $n \to \infty$

In this study, we proposed the polynomial function of the form as follows (Okunuga and Ehijie, 2009):

$$Y(x) = \sum_{j=0}^{M} a_j (x - x_n)^j \tag{6}$$

which is of the type in Equation (3) to develop a continuous LMM for the solution of IVPS of the form:

$$y''' = f(x, y, y', y''),$$
$$y(a) = y_0\ , y'(x) = \partial_0\ ,\ \ y''(a) = \partial_N \tag{7}$$

DERIVATION OF THE METHOD

In this section, we approximate the exact solution $y(x)$ by a polynomial of degree M of the form:

$$Y(x) = \sum_{j=0}^{m} a_j (x - x_n)^j \ \ \cong y(x)\ , \tag{8}$$

$$x_n \leq x \leq x_{n+k}$$

$$Y(x) = a_0 + a_1(x - x_n) + a_2(x - x_n)^2 + a_3(x - x_n)^3 + a_4(x - x_n)^4 + \cdots + a_{m-1}(x - x_n)^{m-1} + a_m(x - x_n)^m \tag{9}$$

Differentiating Equation (9) up to the third-order to obtain

$$Y'''(x) = 6a_3 + 24a_4(x - x_n) + \cdots + (m-1)(m-2)(m-3)a_{m-1}(x - x_n)^{m-4} + (m)\ (m-1)(m-2)a_m(x - x_n)^{m-3} = f(x, y, y', y'') \tag{10}$$

Interpolating Equation 9 at $x = x_n, x_{n+1}, \ldots, x_{n+k-1}$ and collocating Equation 10 at $x = x_n, x_{n+1}, \ldots, x_{n+k}$ we have

$$Y(x_n) = Y_n = a_0$$

$$Y(x_{n+1}) = Y_{n+1} = a_0 + a_1 h + a_2 h^2 + a_3 h^3 + \cdots + a_{m-1} h^{m-1} + a_m h^m$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$Y(x_{n+k-1}) = Y_{n+k-1} = a_0 + (k-1)a_1 h + (k-1)^2 h^2 a_2 + \cdots + a_m (k-1)^m h^m$$

Collocating Equation (10) at $x = x_n, x_{n+1}, \ldots, x_{n+k}$, we have

$$6a_3 = f_n$$

$$6a_3 + 24a_4 h + \cdots + (m-1)(m-2)(m-3)a_{m-1}h^{m-4} + (m)(m-1)(m-2)a_m h^{m-3} = f_{n+1}$$

$$6a_3 + 2.24a_4 h + \cdots + 2^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-4} + 2^{m-2}(m)(m-1)(m-2)a_m h^{m-3} = f_{n+2}$$

$$6a_3 + k.24a_4 h + \cdots + k^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-4} + k^{m-3}(m)(m-1)(m-2)a_m h^{m-3} = f_{n+k}$$

Multiplying the collocation by h^3 yield

$$h^3 6a_3 = h^3 f_n$$

$$h^3 6a_3 + 24a_4 h^4 + \cdots + (m-1)(m-2)(m-3)a_{m-1}h^{m-1} + (m)(m-1)(m-2)a_m h^m = h^3 f_{n+1}$$

$$h^3 6a_3 + 2.24a_4 h^4 + \cdots + 2^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-1} + 2^{m-2}(m)(m-1)(m-2)a_m h^m = h^3 f_{n+2}$$

$$h^3 6a_3 + k.24a_4 h^4 + \cdots + k^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-1} + k^{m-3}(m)(m-1)(m-2)a_m h^m = = h^3 f_{n+k}$$

Hence the matrix equation is given as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & h & h^2 & h^3 & ... & h^{m-2} & h^{m-1} & h^m \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 1 & (k-1)h & (k-1)^2h^2 & (k-1)^3h^3 & ,,, & (k-1)^{m-2}h^{m-2} & (k-1)^{m-1}h^{m-1} & (k-1)^m h^m \\ 0 & 0 & 0 & 6h^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6h^3 & 24h^4 & ... & (m-1)(m-2)(m-3)h^{m-1} & (m)(m-1)(m-2)h^m \\ ... & ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & .... & .... & ... \\ 0 & 0 & 0 & 6h^3 & k.24h^4 & ... & ... & k^{m-3}m(m-1)(m-2)h^m \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \dots \\ \dots \\ a_m \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+1} \\ .. \\ y_{n+k-1} \\ h^3 f_n \\ h^3 f_{n+1} \\ \dots \\ h^3 f_{n+k} \end{bmatrix} \quad (11)$$

The matrix above is solved to obtain the values of $a_j's \ , j = 0(1)m$, which are then substituted into Equation (6).

After some algebraic simplification, we obtained a continuous polynomial, which is then evaluated at $x = x_{n+k}$. The resulting k-step LMM is of the form:

$$\sum_{j=0}^{k} \alpha_j(x) y_{n+j} = h^3 \sum_{j=0}^{k} \beta_j(x) f_{n+j} \quad (12a)$$

SPECIFICATION OF THE METHOD

Let

$$Y(x) = \sum_{j=0}^{m} a_j (x - x_n)^j \quad (12b)$$

as in Equation (6). By letting M = 2K, then

$$Y(x) = \sum_{j=0}^{2K} a_j (x - x_n)^j \ , h = x - x_n \ , k = 4 \quad (13)$$

$$= a_0 + a_1(x - x_n) + a_2(x - x_n)^2 + a_3(x - x_n)^3 + a_4(x - x_n)^4 + a_5(x - x_n)^5 + a_6(x - x_n)^6 + a_7(x - x_n)^7 + a_8(x - x_n)^8 \quad (14)$$

Differentiating Equation (14) up to the third order, we have

$$Y'''(x) = 6a_3 + 24a_4(x - x_n) + 60a_5(x - x_n)^2 + 120a_6(x - x_n)^3 + 210a_7(x - x_n)^4 + 336a_8(x - x_n)^5 \quad (15)$$

Interpolating Equation (13) at $x = x_n, x_{n+1}, x_{n+2}, x_{n+3}$ and collocating Equation (15) at $x = x_n, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}$ to give the system of equation

$$\sum_{j=0}^{2K} \alpha_j (x - x_n)^j = y_{n+j} \quad (16a)$$

$$\sum_{j=3}^{2K} j(j-1)(j-2)\alpha_j (x - x_n)^{j-3} = f_{n+j} \quad (16b)$$

The matrix equation arising from this is given as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & h & h^2 & h^3 & h^4 & h^5 & h^6 & h^7 & h^8 \\ 1 & 2h & 4h^2 & 8h^3 & 16h^4 & 32h^5 & 64h^6 & 128h^7 & 256h^8 \\ 1 & 3h & 9h^2 & 27h^3 & 81h^4 & 243h^5 & 729h^6 & 2187h^7 & 6561h^8 \\ 0 & 0 & 0 & 6h^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6h^3 & 24h^4 & 60h^5 & 120h^6 & 210h^7 & 336h^8 \\ 0 & 0 & 0 & 6h^3 & 48h^4 & 240h^5 & 960h^6 & 3360h^7 & 10710h^8 \\ 0 & 0 & 0 & 6h^3 & 72h^4 & 540h^5 & 3240h^6 & 17010h^7 & 81648h^8 \\ 0 & 0 & 0 & 6h^3 & 96h^4 & 960h^5 & 7680h^6 & 53760h^7 & 334064h^8 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ h^3 f_n \\ h^3 f_{n+1} \\ h^3 f_{n+2} \\ h^3 f_{n+3} \\ h^3 f_{n+4} \end{bmatrix} \quad (17)$$

Equation (17) is solved by Gaussian elimination or matrix inversion to obtain the $a_j's \ , j = 0(1)2k$ which when substituted in Equation (13), we have the continuous polynomial given by:

$$Y(x) = \frac{1}{20160h^8}[20160h^8 + 10560h^7(x - x_n) - 64080h^6(x - x_n)^2 + 80640h^4(x - x_n)^4 - 67200h 23520h^2(x - x_n)^6 - 3840h(x - x_n)^7 + 240(x - x_n)^8]y_n$$

$$+\frac{1}{20160h^8}[-82080h^7(x - x_n) + 202320h^6(x - x_n)^2 + 241920h^4(x - x_n)^4 + 201600h^3(x - x_n)^5 - 70560h^2(x - x_n)^6 + 11520h(x - x_n)^7 - 720(x - x_n)^8]y_{n+1}$$

$$+\frac{1}{20160h^8}[11232h^7(x - x_n) - 212400h^6(x - x_n)^2 + 241920h^4(x - x_n)^4 - 201600h^3(x - x_n)^5 + 70560h^2(x - x_n)^6 - 11520h(x - x_n)^7 + 720(x - x_n)^8]y_{n+2}$$

$$+\frac{1}{20160h^8}[-40800h^7(x - x_n) + 74160h^6(x - x_n)^2 - 80640h^4(x - x_n)^4 + 67200h^3(x - x_n)^5 - 23520h^2(x - x_n)^6 + 3840h(x - x_n)^7 - 240(x - x_n)^8]y_{n+3}$$

$$+\frac{1}{20160h^8}[1398h^{10}(x - x_n) + 4200h^9(x - x_n)^2 - 3360h^8(x - x_n)^3 - 1414h^7(x - x_n)^4 + +210h^6(x - x_n)^5 + 28h^5(x - x_n)^6 - 12h^4(x - x_n)^7 + h^3(x - x_n)^8]f_n$$

$$+\frac{1}{20160h^8}[26064h^{10}(x - x_n) - 44328h^9(x - x_n)^2 + 42336h^7(x - x_n)^4 + -33936h^6(x - x_n)^5 + 11620h^5(x - x_n)^6 - 1872h^4(x - x_n)^7 + 116h^3(x - x_n)^8]f_{n+1}$$

$$+\frac{1}{20160h^8}[20052h^{10}(x-x_n)-36330h^9(x-x_n)^2+39816h^7(x-x_n)^4+$$
$$-33684h^6(x-x_n)^5+12012h^5(x-x_n)^6-1992h^4(x-x_n)^7+126h^3(x-x_n)^8]f_{n+2}$$
$$+\frac{1}{20160h^8}[-48h^{10}(x-x_n)+88h^9(x-x_n)^2-224h^7(x-x_n)^4+$$
$$336h^6(x-x_n)^5-196h^5(x-x_n)^6+48h^4(x-x_n)^7-4h^3(x-x_n)^8]f_{n+3}$$
$$+\frac{1}{20160h^8}[54h^{10}(x-x_n)-99h^9(x-x_n)^2+126h^7(x-x_n)^4+$$
$$-126h^6(x-x_n)^5+56h^5(x-x_n)^6-12h^4(x-x_n)^7+h^3(x-x_n)^8]f_{n+4} \quad (18)$$

Evaluating Equation (18) at $x = x_{n+4}$ yield the following discrete:

$$y_{n+4}-2y_{n+3}+2y_{n+1}-y_n=\frac{h^3}{120}(f_{n+4}+56f_{n+3}+126f_{n+2}+56f_{n+1}+f_n) \quad (19)$$

ANALYSIS AND IMPLEMENTATION OF THE METHOD

The method Equation (19) is a specific member of the conventional LMM which can be expressed as

$$\sum_{j=0}^{k}\alpha_j y_{n+j}=h^3\sum_{j=0}^{k}\beta_j y'''_{n+j} \quad (20)$$

This can be written symbolically as:

$$\rho(E)y_n-h^n\sigma(E)=0, \quad f_n=f(x_n,y_n) \quad (21)$$

E is the shift operator defined as $E^{\,i}y_n=y_{n+j}$ and $\rho(E)$ and $\sigma(E)$ are respectively the first and second characteristics polynomial of the LMM defined as:

$$\rho(E)=\sum_{j=0}^{k}\alpha_j E^j, \quad \alpha_k\neq 0 \quad ,\sigma(E)=\sum_{j=0}^{k}\beta_j E^j \quad (22)$$

Following Fatunla (1988) and Lambert (1973), we define the local truncation error associated with Equation (20) by the difference operator

$$L[y(x);h]=\sum_{j=0}^{k}[\propto_j y(x_n+jh)-h^3\beta_j y'''(x_n+jh)] \quad (23)$$

where $y(x)$ is assumed to have continuous derivatives of sufficiently high order. Therefore, expanding Equation (23) in Taylor series about the point x to obtain the expression

$$L[y(x);h]=c_0y(x)+c_1hy'(x)+c_2h^2y''(x)+\cdots+c_{p+2}h^{p+2}y^{p+2}(x) \quad (24)$$

where the $c_0,c_1,c_2,c_3,\ldots,c_p,\ldots,c_{p+2}$ are defined as

$$C_0=\sum_{j=0}^{k}\alpha_j$$

$$C_1=\sum_{j=1}^{k}j\alpha_j$$

$$C_2=\frac{1}{2!}\sum_{j=1}^{k}j^2\alpha_j$$

$$C_q=\frac{1}{q!}\left[\sum_{j=1}^{k}j^q\alpha_j-q(q-1)(q-2)\sum_{j=1}^{k}\beta_j j^{q-3}\right]$$

In the sense of Lambert (1973), we say that the method of Equation (20) is of order p and error constant C_{p+3} if

$$C_0=C_1=C_2=\cdots=C_p=C_{p+1}=C_{p+2}=0, \quad C_{p+3}\neq 0$$

Using the concept above, the method Equation (19) has order $p=7$ and error constant given by:

$$c_{p+2}=\frac{12}{362880}$$

In order to analyze the method for zero stability we write:

$$\rho(r)=r^4-2r^3+2r-1,$$

Where $r-1$ is a factor

Therefore we have

$$(r-1)(r^3-r^2-r+1)=(r-1)(r-1)(r^2-1)$$

And $r=1,1,1$ and 1 so that $|r|\leq 1$. Thus the method is zero stable

IMPLEMENTATTION

Single step method can be used to solve higher order ordinary differential equations directly without the need to first reducing it to an equivalent system of first order.

Consider the IVPS in Equation (7). For our method of order p = 7, Taylor series expansion is used to calculate

$y_{n+1}, y_{n+2}, y_{n+3}$, and their first, second, third derivatives up to order p = 7

$$y_{n+j}\equiv y(x_n+jh)\cong y(x_n)+jhy'(x_n)+\frac{(jh)^2}{2!}y''(x_n)+\frac{(jh)^3}{3!}f_n+\frac{(jh)^4}{4!}f'_n+\frac{(jh)^5}{5!}f''_n+\frac{(jh)^6}{6!}f'''_n+\frac{(jh)^7}{7!}f^{iv}_n\ldots$$

Table 1. Results of test Problem 1.

X-value	Exact solution	New result (p = 7)	Error in Adesanya (2011) Block method (P = 7)	Error in our new method (P = 7)
0.1	4.98752E-003	4.98752E-003	1.189947E-11	1.1899E-11
0.2	1.98011E-002	1.98011E-002	3.042207E-09	3.0422E-09
0.3	4.39996E-002	4.39997E-002	7.779556E-08	7.7796E-08
0.4	7.68675E-002	7.68676E-002	7.746692E-07	1.5559E-07
0.5	1.174433E-00	1.174437E-00	4.59901E-06	3.0541E-07
0.6	1.645579E-001	1.64558E-001	6.478349E-06	4.6102E-07
0.7	2.16881E-001	2.16881E-001	5.783963-06	3.138E-07
0.8	2.72975E-001	2.72976E-001	2.354715E-06	7.0374E-07
0.9	3.31350E-001	3.31352E-001	3.766592E-06	1.0177E-06
1.0	3.905280E-001	3.90531E-001	1.23312E-05	1.6528E-06

$$y_{n+j}^{(1)} \equiv y'(x_n + jh) \cong y'(x_n) + jhy''(x_n) + \frac{(jh)^2}{2!} f_n + \frac{(jh)^3}{3!} f'_n + \frac{(jh)^4}{4!} f''_n + \frac{(jh)^5}{5!} f'''_n + \frac{(jh)^6}{6!} f_n^{iv} + \frac{(jh)^7}{7!} f_n^{v} \ldots$$

$$y_{n+j}^{(2)} \equiv y''(x_n + jh) \cong y''(x_n) + jhf_n + \frac{(jh)^2}{2!} f'_n + \frac{(jh)^3}{3!} f''_n + \frac{(jh)^4}{4!} f'''_n + \frac{(jh)^5}{5!} f_n^{iv} + \frac{(jh)^6}{6!} f_n^{v} + \frac{(jh)^7}{7!} f_n^{vi} \ldots$$

$$y_{n+j}^{(3)} \equiv y'''(x_n + jh) \cong f_n + jhf'_n + \frac{(jh)^2}{2!} f''_n + \frac{(jh)^3}{3} f'''_n + \frac{(jh)^4}{4!} f_n^{iv} + \frac{(jh)^5}{5!} f_n^{v} + \frac{(jh)^6}{6!} f_n^{vi} + \frac{(jh)^7}{7!} f_n^{vii} \ldots$$

Then the known values of x_n and y_n are substituted into the differential equations. Next the differential equation is differentiated to obtain the expression for higher derivatives using partial differentiation as follows:

$$y''' = f(x, y, y', y'') \qquad = f_j$$

$$y^{iv} = f_x + y' f_y + y'' f_{y'} + f f_{y''} = \left(\frac{\partial}{\partial x} + y' \frac{\partial}{\partial y} + y'' \frac{\partial}{\partial y'} + f \frac{\partial}{\partial y''}\right) f_j = Df_j$$

$$y^{v} = f_{xx} + (y')^2 f_{yy} + (y'')^2 f_{y'y'} + f^2 f_{y''y''} + 2y' f_{xy} + 2y'' f_{xy'} + 2f f_{xy''} + 2y'y'' f_{yy'} + 2y' f f_{yy''} + 2y'' f f_{y'y''} + Df_j(f_{y''}) + f_j(y'' + f_{y'})$$

$$= D^2 f_j + (f_{y''}) Df_j + f_j (y'' + f_{y'})_j,$$

where

$$D = \frac{\partial}{\partial x} + y' \frac{\partial}{\partial y} + y'' \frac{\partial}{\partial y'} + f \frac{\partial}{\partial y''}, \text{ and } D^2 = D(D)$$
$$\vdots$$

$D^p f_j$ where p is the order of the method.

Numerical experiments

Our methods of order p = 7 were used to solve some IVPS of both general and special nature using Taylor's series. Our results were compared with the results of other researchers in this area.

The following IVPS were used as test problems:

1) $y''' + 4y' = x\ , y(0) = 0, y'(0) = 0, y''(0) = 1, h = 0.1, 0 \le x \le 1$

Exact solution: $y(x) = \frac{3}{16}(1 - cos2x)$,

2) $y''' = -y, y(0) = 1, y'^{(0)} = -1, y''^{(0)} = 1,\ 0 \le x \le 1,\ h = 0.1$

Exact solution: $y(x) = e^{-x}$,

3) $y''' + y'' + 3y' - 5y = 2 + 6x - 5x^2, y(0) = -1, y'(0) = 1, y''(0) = -3,\ 0 \le x \le 1$

Exact solution: $y(x) = x^2 - e^x + e^{-x} \sin(2x)$.

The results of these problems were compared with that of Adesanya (2011), Awoyemi (2003) and Olabode (2007) which were implemented in predictor corrector mode and block method, respectively.

The error in this experiment is defined as:

$$Error = |y(x) - y_n(x)|,$$

Where y(x) is the exact solution and $y_n(x)$ is the computed result.

Conclusion

We have developed a k-step LMM using power series collocation method. A new scheme with continuous coefficient is obtained which was applied to solve some special and general third-order IVPS in ordinary differential equation. Evidence of the better accuracy of our method over existing methods are mentioned in Tables 1, 2 and 3.

Table 2. The result of test Problem 2.

X-value	Exact solution	New result (P =7)	Error in Olabode (2007) (P = 7)	Error in our new method (P = 7)
0.1	0.904837	0.940837	1.36929E-09	2.4525E-13
0.2	0.818731	0.818731	3.12272E-08	6.2109E-11
0.3	0.740818	0.740818	1.27694E-07	1.5746E-10
0.4	0.670320	0.670320	3.25196E-07	3.1477E-09
0.5	0.606531	0.606531	6.54297E-07	6.1617E-09
0.6	0.548812	0.548812	1.14406E-06	9.1732E-09
0.7	0.496585	0.496585	1.81784E-06	1.3329E-08
0.8	0.449329	0.449329	2.69774E-06	1.6378E-08
0.9	0.406570	0.406570	3.80241E-06	1.7134E-08
1.0	0.367879	0.367879	5.14755E-06	7.4405E-09

Table 3. The result of test Problem 3.

X-value	Exact solution	New result (P =7)	Error in Olabode (2007) (P = 7)	Error in our new method (P = 7)
0.1	0.915407E+00	0.915407E+00	6.408641E-07	8.54782E-11
0.2	0.862574E+00	0.862574E+00	1.511330E-05	2.232510E-09
0.3	0.841561E+00	0.841561E+00	6.364443E-05	5.824412E-08
0.4	0.850967E+00	0.850967E+00	1.675667E-04	1.226405E-06
0.5	0.888343E+00	0.888341E+00	3.507709E-04	2.811820E-06
0.6	0.950606E+00	0.950599E+00	6.410875E-04	6.295841E-06
0.7	0.103439E+01	0.103438E+01	1.071642E-03	1.695782E-05
0.8	0.113640E+01	0.113636E+01	1.682213E-03	4.765221E-05
0.9	0.125367E+01	0.125353E+01	2.520603E-03	1.316541E-04
1.0	0.138377E+0	0.138343E+0	3.644014E-03	3.417856E-04

REFERENCES

Adeniyi RB, Alabi MO (2006). Derivation of continuous multistep methods using Chebyshev polynomial basis functions. Abacus 33:351-361.

Adesanya AO (2011). Block methods for direct solutions of general higher order initial value problems of ordinary differential equations: PhD thesis submitted to the Department of mathematical sciences, the Federal University of Technology, Akure.

Awoyemi DO (1999). A class of continuous methods for the solution of general second order ordinary differential equations. Inter. J. Comput. Math. 72:29-37.

Awoyemi DO (2003). A P-stable linear Multistep method for solving general third order ordinary differential equations. Int. J. Comput. Math. 80(8):987-993.

Bruguano L, Trigiante D (1998). Solving differential Problems by Multistep Initial and Boundary Value Methods, Gordon and Breach Science Publishers, Amsterdam, pp. 280-299.

Butcher JC (2003). Numerical methods for ordinary differential equations, John Wiley & Sons, New York.

Fatunla SO (1988). Numerical Methods for initial value problems in ordinary differential Equations, Academic Press Inc. Harcourt Brace, Jovanovich Publishers, New York.

Henrici P (1962). Discrete Variable Method in Ordinary differential Equations, John Wiley & Sons, New York.

Lambert JD (1973). Computional Methods in ODEs. John Wiley & Sons, New York.

Olabode BT (2007). Some linear multistep methods for special and general third order initial value problems in ordinary differential equations. Ph.D thesis in the department of Mathematical Sciences, The Federal University of Technology, Akure.

Onumanyi P, Oladele JO, Adeniyi RB, Awoyemi DO (1993). Derivation of Finite difference method by collocation. Abacus 23(2):72-83.

Okunuga SA, Ehijie J (2009). New Derivation of continuous multistep methods using power series as basis function. J. Modern Math. Stat. 3(2).

academicJournals

Related Journals Published by Academic Journals

■African Journal of Pure and Applied Chemistry

■Journal of Internet and Information Systems

■Journal of Geology and Mining Research

■Journal of Oceanography and Marine Science

■Journal of Environmental Chemistry and Ecotoxicology

■Journal of Petroleum Technology and Alternative Fuels